Durbin- Watson Test for detecting auto-correlation in the regression model.

The simplest and most commonly used model is one where the errors u_t and u_{t-1} have a correlation ρ. For this model we can think of testing hypothesis about ρ on the basis $\hat{\rho}$, the correlation b/w the least squares residuals $\hat{u}_t$ and $\hat{u}_{t-1}$. A commonly used statistic for the purpose is the Durbin Watson (DW) statistic, which we will denote by d. It is defined as,

$$d = \frac{\sum_{2}^{n} \left(\hat{u}_t - \hat{u}_{t-1}\right)^2}{\sum_{1}^{n} \hat{u}_t^2}$$

where $\hat{u}_t$ is the estimated residual for period t. We can write d as,

$$d = \frac{\sum \hat{u}_t^2 + \sum \hat{u}_{t-1}^2 - 2\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}$$

Since $\sum \hat{u}_t^2$ and $\sum \hat{u}_{t-1}^2$ are approximately equal if the sample is large, we have

$d \simeq 2(1-\hat{\rho})$.

$$d = \frac{2\sum \hat{u}_t^2 - 2\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}$$

$$= 2\left(1 - \frac{\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2}\right)$$

$$\frac{\sum \hat{u}_t \hat{u}_{t-1}}{\sum \hat{u}_t^2} = \hat{\rho}$$

Therefore, we get,

$$d \simeq 2(1-\hat{\rho}).$$

If $\hat{\rho} = +1$ then $d = 0$ and if $\hat{\rho} = -1$ then $d = 4$. We have $d = 2$ if $\hat{\rho} = 0$. If d is close to 0 or 4 the residuals are highly correlated

The sampling distribution of d depends on the values of the explanatory variables and hence Durbin-Watson derived upper (d_u) limits and lower (d_L) limits for the significance level for d. There are tables to test the hypothesis of zero auto-correlation against the hypothesis of 1st order autocorrelation. (For negative auto correlation we interchange d_L and d_u).

(i) If $d < d_L$ we reject the H_0 of no autocorrelation

(ii) If $d > d_u$ we do not reject the H_0

(iii) $d_L < d < d_u$ the test is inconclusive

The significance points in the DW tables at the end of the book are tabulated for testing $\rho = 0$ against $\rho > 0$. If $d > 2$ and we wish to test the hypotheses $\rho = 0$ against $\rho < 0$ we consider $(4-d)$ and refer to the durbin-Watson tables as if we are testing for positive auto-correlation.

Although, we have said that $d \simeq 2(1-\hat{\rho})$ this approximation is valid only in large samples. The mean of d when $\rho = 0$ has been shown to be given approximately by

$$E(d) \simeq 2 + \frac{2(k-1)}{(n-k)}$$

where k is the number of regression parameters estimated including the constant term and n is the large sample size.